Towards Concise Models of Grid Stability

Vadim Arzamasov
Karlsruhe Institute of Technology
Karlsruhe, Germany
vadim.arzamasov@kit.edu

Klemens Böhm
Karlsruhe Institute of Technology
Karlsruhe, Germany
klemens.boehm@kit.edu

Patrick Jochem
Karlsruhe Institute of Technology
Karlsruhe, Germany
patrick.jochem@kit.edu

***Abstract*— Decentral Smart Grid Control (DSGC) is a new system implementing demand response without significant changes of the infrastructure. While models of DSGC exist, they rely on various simplifying assumptions. For example, researchers have assumed that the behavior of all participants in the grid is identical. In this paper we study how data-mining techniques can help to remove some of these simplifications, while keeping the representation of the insights gained concise. We systematically collect the various assumptions and identify questions regarding the system that are still open. Next, we run many simulations, with diverse input values. Finally, we apply decision trees to the resulting data and demonstrate that this does indeed give way to new insights. For example, we discover that the system can be stable even if some participants adapt their energy consumption with a high delay, a finding that goes beyond previous results, or that fast adaptation is preferable for system stability.**

***Keywords*— Simulations, metamodeling, electricity grid**

I. Introduction

Motivation. Electrical grids require a balance between electricity supply and demand in order to be stable. Conventional systems achieve this balance through demand-driven electricity production. For future grids with a high share of inflexible (i.e., renewable) energy sources however, the concept of demand response is a promising solution. This implies changes in electricity consumption in reaction to electricity-price changes. There are different ways to define the price and communicate it to consumers, including local electricity auctions or deriving the price through building reliable demand and supply forecasts. The Decentral Smart Grid Control (DSGC) system, proposed recently [1], [2], has received much attention. This is because it does not require collecting and processing large amounts of data. It ties the electricity price to the grid frequency so that it is available to all participants, i.e., all energy consumers and producers.

Current models of DSGC come together with assumptions. Some assumptions facilitate simulations of its stability in [1], [2], i.e., to infer whether the behavior of participants in response to price changes destabilizes the grid. To do so, the system is described with differential equations. Following [3], we refer to the variables of the equations as *inputs*. The current approach of studying stability consists of two steps:

(1) Assign fixed values to some inputs across all equations and simulation runs.
(2) For the other inputs, draw values from a fixed distribution in each experiment (equal across equations).

The result is a set of one-dimensional intervals describing the dependence of stability on tabulated input values [1], [2]. For example, the inference from such an analysis carried out in [2] could look as follows. (We omit some inputs and units to keep the example simple.)

If $T_j \equiv T = 2$ *and* $\gamma_j \equiv \gamma = 0.25$ *and* $\tau_j \equiv \tau \in [0, 0.7] \cup [2.2, 5]$ *then the system is stable*

We see two shortcomings of this methodology. First, changing only one input value at a time as in the example, with only input τ varying, leads to the consideration of only a few values of the other inputs (T and γ). This does not facilitate any estimation of interactions among inputs [3]. We refer to this problem as the *fixed inputs issue*. Second, the assumption of equal input values is not realistic, especially when they are inherent to system participants, e.g., private households, and cannot be regulated externally. Examples of these inputs are price elasticities of energy consumers, i.e., their willingness to change consumption in response to price changes, or the time they need to react to such a change. We call this *equality issue.*

In this paper we study how data-mining methods can help to solve these issues. We seek a method to analyze the DSGC system for many diverse input values, removing those restrictive assumptions on input values. In other words, our objective is to create metamodels. We want to keep these metamodels simple, hence the word "concise" in the title of this article. – At the same time, we seek new insights regarding the behavior of the DSGC system.

Challenges. To deal with the fixed inputs issue, one might allow simultaneous changes of several input values in the simulations. This leads to the question in which form the results of such an analysis should be presented so that they remain comprehensible. An example of this representation for the system could be:

If $T_j \equiv T \in [2.5, 4]$ *and* $\gamma_j \equiv \gamma \in [0, 0.5]$ *and* $\tau \in [1, 3.7]$, *then the system is stable*

The difference to the first example is that now all conditions are intervals.

A solution to the equality issue would be to allow input values to be different from each other and to be drawn from some reasonable distribution. But this is not trivial. Think of a system described by four equations, each having three inputs, which already leads to $4 \cdot 3 = 12$ degrees of freedom. This means that in general there will be intervals for all 12 inputs in the description of stability regions, as follows:

If $T_1 \in [2.5, 4]$ *and* $T_2 \in [1.7, 3]$ *and* ..., *and* $\tau_3 \in [2.2, 4.1]$ *and* $\tau_4 \in [1, 3.7]$ *then the system is stable*

This work was supported by the German Research Foundation (Deutsche Forschungsgemeinschaft), Research Training Group 2153: "Energy Status Data — Informatics Methods for its Collection, Analysis and Exploitation"

Clearly, comprehensibility suffers. The problem is even more prominent with more inputs.

Contributions. To identify current limitations of the DSGC, we systematically collect the assumptions behind it. We have already described two, the fixed inputs and the equality issue. But as we will show, there are more. To our knowledge, despite the rising popularity of this system, a respective comprehensive summary in the literature does not exist.

To deal with the fixed inputs and equality issues, we investigate system stability for different design points and apply one specific data-mining method, namely decision trees, to the results. To deal with the many inputs and to make the description of stability regions more comprehensible, we replace the inputs of the system with aggregates, referred to as *features.* An example rule in the feature space is:

If $avg(T_j) \in [2.5, 4]$ *and* $min(\gamma_j) < 0.25$ *and* $max(\tau_j) > 3$, *then the system is stable*

We demonstrate that the approach indeed gives way to new insights into the simulated system. For instance, we have learned that fast adaptation generally improves system stability.

Paper outline. Section 2 reviews related work. Section 3 lists the assumptions behind the DSGC model. Section 4 describes our methodology. Section 5 features the results. Section 6 discusses future work. Section 7 concludes.

II. Literature Review

In this section we first review work comparing comprehensibility of different statistical models as well as articles applying such models to simulation data. Then we describe how machine learning (ML) models have been used to study stability of the electricity grid.

A. Concise Metamodels

Several studies suggest that results produced by decision trees or classification rules are quite comprehensible [4], [5], as compared to other data-mining methods. Examples of using rules as metamodels are in [6], [7], [8] and [9]. In [9] and [7], the GENREG rule learning program is applied to simulations in manufacturing. [8] uses a specific heuristic to construct oblique rules in the same domain. [6] features a fuzzy-rule model and its application to data simulated from a token-bus model. The primary target of most papers has been a new rule-learning algorithm rather than exploring a specific system.

B. ML Models for Electricity Grid Stability

There is a number of studies applying ML models to analyze the security/stability of power systems. [10] applies a neural network to security problems within a power system in California. System behavior is studied in 1792 simulation runs. The resulting boundary has been visualized in the form of so-called nomograms. This form of visualization shares some disadvantages with the approach described in Section 1. Specifically, it considers two inputs at a time and hence does not explicitly support exhaustive insights with more inputs. [11] proposes modelling a non-linear security boundary by using features formed as monomials of the original input up to a certain degree. They solve the problem of a large number of features with kernel ridge regression. Two systems are simulated: the New England 39-bus system and a larger 470-bus system. [12] applies support vector machines and neural networks to analyze the transient stability of a 2484-bus system. The data is generated in 1242 simulation runs and contains 244 inputs. To reduce the number of inputs, a feature selection mechanism is applied in some cases. This work concludes that support vector machines with no feature selection outperform neural networks with feature selection in terms of accuracy. In [13] decision trees are used to study the transient stability of a toy 9-bus system and of the 1696-bus Iran national grid. The authors then compare the results to those obtained with ANN and SVM models. In all cases the accuracy is about 99% although it is not clear whether the out-of-sample data was used for testing. – All these studies target at accurate prediction, but not at a general view on the system.

III. Decentral Smart Grid Control (DSGC)

In this section we first review the system under consideration proposed in [1], including the derivation of the physical model and its economic superstructure. Next, we systematically list assumptions and open issues regarding the DSGC. Some assumptions are inherent to the derivation of equations describing the system. One cannot remove the assumptions without changing the equations. Examples are (5), (6) and (9) below. The other assumptions are ones made when analyzing the system with simulations. An example is that some inputs have certain fixed values. In this study, we want to loosen these restrictions by removing assumptions of the second type.

A. The Model

The DSGC system consists of two parts. The first, physical part describes the dynamics of generators and loads based on the equations of motion [14], [15]. The second part is an economic superstructure binding the electricity price to the grid frequency, proposed and studied in [1], [2].

1) Physical model.

Both generators and loads are modelled as rotating machines. Energy conservation laws define the dynamics of a respective system as follows:

$$P^{source} = P^{accumulated} + P^{dissipated} + P^{transmitted} \tag{1}$$

That is, the power generated is accumulated in the rotational motion of the generator or dissipated or transmitted to the loads. Replacing terms in (1) with respective equations yields:

$$P_j^{source} = \frac{1}{2} M_j \frac{d}{dt} (\dot{\delta}_j)^2 + \kappa_j (\dot{\delta}_j)^2 - \sum_k P_{jk}^{max} \sin(\delta_k - \delta_j) \tag{2}$$

where j is an index of the system participant (load or generator), M is the moment of inertia, κ is a friction coefficient, P_{jk}^{max} is the capacity of the line connecting Participants j and k. $\delta_j(t)$ is rotor angle (or phase). [1] then specifies:

$$\delta_j(t) = \omega t + \theta_j(t) \tag{3}$$

where ω is a grid reference frequency (e.g., 50 Hz) and $\theta_j(t)$ is a rotor angle relative to it. Finally, (3) is substituted in (2) to yield:

$$\frac{d^2\theta_j}{dt^2} = P_j - \alpha_j \frac{d\theta_j}{dt} + \sum\nolimits_k K_{jk} \sin(\theta_k - \theta_j) \tag{4}$$

TABLE I. INPUTS OF DSGC

Input	Description
P_j	Mechanical power produced/consumed
α_j	Damping constant
K_{jk}	Coupling strength, proportional to line capacity
γ_j	Coefficient, proportional to price elasticity
τ_j	Reaction time, the delay between a price change and adaptation to it
T_j	Averaging time, required to measure price signal

where we abbreviate $K_{jk} = \left(P_{jk}^{max}\right)/\left(M_j\omega\right)$, $\alpha_j = \left(2\kappa_j\right)/M_j$ and $P_j = \left(P_j^{source} - \kappa_j\omega^2\right)/M_j$. For the transition from (2) to (4) to be correct, the following assumptions must hold [14]:

$$d\theta_j/dt \ll \omega \quad (5)$$

$$d^2\theta_j/dt^2 \ll 2\kappa_j\omega/M_j \quad (6)$$

2) Economic superstructure

The idea of [1] is binding the electricity price to the grid frequency, with some proportionality factor c_1, and letting participants adjust their production/consumption with price changes. The additional equations are:

$$p_j = p_\omega - c_1 \cdot \int_{t-T_j}^{t} \frac{d\theta_j}{dt}(t-\tau_j)dt \quad (7)$$

$$\hat{P}_j(p_i) \approx P_j + c_j(p_j - p_\omega) \quad (8)$$

where p_j is the electricity price for the j-th participant, $\hat{P}_j$ is the power consumed/produced at price p_j, c_j is a coefficient proportional to the price elasticity, p_ω is the electricity price when $d\theta_j/dt \equiv 0$, τ_j is the reaction time, i.e., the time after which one adjusts consumption/production to a price change, T_j is so-called averaging time: The average frequency during Period T_j defines the price . It is assumed that:

$$\sum P_j \equiv 0 \quad (9)$$

The final equation describing the dynamics of the DSGC results from substituting P_j in (4) with $\hat{P}_j$ from (8), where p_j is defined as in (7), and denoting $\gamma_j = c_1 \cdot c_j$.

$$\frac{d^2\theta_j}{dt^2} = P_j - \alpha_j\frac{d\theta_j}{dt} + \sum_{k=1}^{N} K_{jk}\sin\left(\theta_k - \theta_j\right) - \\ -\frac{\gamma_j}{T_j}\Big(\theta_j\left(t-\tau_j\right) - \theta_j\left(t-\tau_j-T_j\right)\Big) \quad (10)$$

Table I summarizes the system inputs.

B. Model Assumptions and Open Questions

In addition to the assumptions in (5), (6) and (9), we now list the implicit ones. We split the assumptions into three parts, one related to the physical equations, one to the equations describing the economic superstructure and one to the way the system has been studied previously. Additionally, we identify two open questions regarding the system.

1) Assumptions behind the Physical Model

1. Generators are modelled as rotating mass. However, the model is often motivated (cf. [1], [14], [15]) by assuming an increasing share of renewables, some of which (PV or wind turbines) do not have any rotational inertia [16].
2. Loads are modelled as rotating mass (synchronous motors)
   a. According to [17], in power engineering "in many of the applications … passive loads are considered instead of motors...". This is particularly true for modelling households [18].
   b. According to [19], [20], "Most of motor loads in the US are indeed induction motors, not synchronous motors"
3. The model completely neglects any so-called control, for instance active control, as stated in [14].
4. The model assumes constant voltages and constant mechanical power. This limits its validity to short time intervals of around 1s [21]. However, the model has been explored to study system dynamics during tens of seconds [1], [2], [14]. The constant mechanical power also contradicts the intermittent character of wind [22] and solar power plants.
5. Treating variable values as constants in (10) implies constant moments of inertia M_j for each participant. However, the inertia in systems with a high penetration of renewables has high variations [16].

2) Assumptions behind the Economic Superstructure

1. Adapting the energy consumption to price change does not change the inertia of the system nor the damping constant. This requires further elaboration on how adaptation takes place. For example, if a consumer switches off some device in response to a price change, this generally does have an effect on inertia and the friction in the system.
2. The adaptation of consumption and production happens permanently. That is, load or generation profiles smoothly oscillate with periods of a couple of seconds, see Fig. 3 in [1]. Smooth consumption behavior again raises the question regarding the mechanism of its adjustment, see Item 1.
3. Consumers do not learn from the past. This means that after a few oscillations of price (and grid frequency), the equilibrium level of production or consumption becomes obvious. The better strategy than continuing to adapt to the current price might be to consume or produce at that level. Additionally, resonances often destabilize the system. This means that consumers use more energy when the price is high and less when it is low. This is not rational.

3) Assumptions Made for Analysis

1. The values of inputs P_j, γ_j and K_{jk} are fixed.
2. The values of inputs are equal for all participants: $X_j = X$, where X_j stands for any input in (10), except for P_j, which must satisfy (9).

4) Open Questions

1. The inventors of the model do not make any statement regarding the scale of model validity. It is unclear to what extent the model is suitable to describe a large country-wise or a small-island grid. In the other words, should every household be modelled as a separate consumer, or can one consumer represent a bigger unit, e.g., a town?
2. When the reaction of the system to disturbance is analyzed, the dynamics of rotor angles are different for each participant in the stabilization period after disturbance. This means that the prices also are different; see (7). Then it is

unclear whether the amount of money paid by consumers equals the income of producers.

In what follows we remove assumptions from Subsection 3) and discuss the new insights from this generalization.

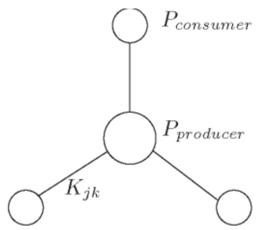

Fig. 1. System structure

IV. Methodology

In this section we first review which input values have been used for DSGC simulations so far and justify our choice of input values, select the data mining model and say how we convert inputs into features. We consider different notions of DSGC stability from literature and select one for further usage.

A. Input Values

We simulate the four-node star electrical grid with centralized production (Fig. 1), studied in [2] as well. A generator is in the center, and three consumers are connected to it.

TABLE II. Ranges of Input Variables

Input	[1], [2], [15]	[14]	Our choice
P_j [a]	$-1s^{-2}$	$-\{1.5, 2, 2.5\}s^{-2}$	$-[0.5, 2]s^{-2}$
α_j	$0.1s^{-1}$	$[0.1, 1]s^{-1}$	$0.1s^{-1}$
K_{jk}	$\{4, 8\}s^{-2}$	$\{5, 7, 8, 10, 12\}s^{-2}$	$8s^{-2}$
γ_j	$0.25s^{-1}$	–	$[0.05, 1]s^{-1}$
τ_j [b]	$[0, 10]s$	–	$[0.5, \{5, 10\}]s$
T_j [b]	$\{0, 1, 2, 4\}s$	–	$2s$ or $[0,4]s$

a. Power consumed by each load (consumer). Power produced by producer is calculated according to (9)

b. In our experiments below, we specify or choice of the upper bound for τ_j and the range for T_j

DSGC system (10) contains six inputs in total. They must be initialized to launch the simulations. The values used in the literature are in Table II. We use the notation $[a, b]$ if the input values are sampled from the interval and $\{a, b\}$ if only values a, b are considered. According to (2)–(4), $K_{jk} = K$ implies that $M_j = M$, i.e., equal moments of inertia for all participants.

For our analysis, we change the values of some inputs and allow others to take values from a defined range instead of a fixed value. To choose which inputs to vary, we classify them as environmental or control, following [23]. For control inputs, an engineer or system designer can set the values. Here, averaging time T_j, damping constant α_j and line capacities K_{jk} are of this type. Environmental (noise) input values depend on the specific user or environment at the time the item is used. Consumed power P_j or reaction time τ_j are environmental. The input γ_j is of mixed type, since in $\gamma_j = c_1 \cdot c_j$ the term c_1 (connecting price to grid frequency) is controllable, and c_j (promotional to price elasticity of participant) is environmental. We fix the values of control inputs and allow the values of the environmental inputs to vary. We also remove the assumption of indistinguishable participants by choosing the input values for each of them independently. More specifically, we make the following changes. The nominal power and coefficients γ_j now take values from a range. We have chosen the range of γ_j in line with [24]. Reaction times are no longer equal among participants. The last column of Table II summarizes this.

B. Model and Experimental Design

Among other machine learning models, decision trees yield results in the form we target at, exemplified in Section I. Earlier results [4], [5] confirm that this form of results is comprehensible. To learn decision trees we choose one of the most popular algorithms, CART [25].

Having defined the ranges of input values and the model, the question arises for which values exactly one should run experiments, i.e., which experimental design to use. Although there is the opinion that one should choose the experimental design together with the statistical model [26], [27], we are not aware of any proof of this being superior for decision trees or classification rules. Intuitively, a space-filling design should be reasonable. We stick to random LHS design [28], [29].

Since the system has symmetries, we hypothesize that a more concise representation of simulation results is feasible based on input aggregates, i.e., features. To create features, we take the minimum, maximum and mean values across all N participants of each input, e.g., $\min \tau_j$ for $j = 1, \dots, N$.

C. Stability Analysis

There are several types of stability of a system [2]. We now briefly describe them and list their advantages and limitations.

1) Stability against Single Perturbations

Here one studies the ability of the system to reach an equilibrium state after some perturbation. It can be specified in terms of power, when some loads require more power for a short time [14]. This type of stability analysis introduces many new degrees of freedom to the simulations:

- Which nodes of the grid to perturb?
- How exactly does the perturbation look like? For instance, in case of a power perturbation, what are its shape, magnitude and time duration?
- How long should one observe a system after the perturbation to draw conclusions on its stability?

2) Basin Stability

To study basin stability, one specifies a range of possible perturbations and simulates the system for a set of randomly sampled perturbations from this range. Next, one may estimate the 'basin volume' as the ratio of initial conditions converging to a stable operation over the total number of initial conditions [1], [2]. Basin stability is more general than stability against perturbation, inheriting all limitations of the latter.

3) Local Stability Analysis (Linear Stability)

Linear stability analysis explores dynamical stability around the steady-state operation of the grid. It consists of finding roots of the characteristic equation, written as:

$$\det A = 0 \tag{11}$$

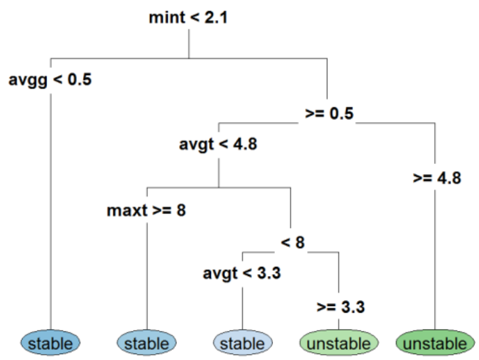

Fig. 2. Excerpt of decision tree for the rebound effect

A is a $2N \times 2N$ matrix derived from equations of motion (10). The equation has infinitely many solutions, but only a finite number of solutions can have a positive real part, and they determine the instability of the system [2]. To find these roots, a numerical optimization problem is solved. This analysis does not have the limitations of the previous two, but other ones:

- There is no guarantee of correct inference from any individual simulation.
- Each element of A has up to n terms, and the determinant operation has a complexity of $O(n!)$. This makes this analysis unsuitable for large systems (> 10 nodes) with heterogeneous participants.

We will use local stability analysis, since it does not bring additional degrees of freedom to an already complex system.

V. Experimental Results

We now evaluate the usefulness of the proposed approach. To do so, we graph the results of applying the CART algorithm to data from our simulations. The paths to the leaves of the decision tree produced by CART define the stable/unstable regions. To avoid clutter in the plots we slightly change the notation, replacing γ_j with g and τ_j with t. In the next subsections we first explore the so-called rebound effect and then show how one can use our approach to set the values for control inputs. At the end of each subsection we discuss new insights which previous analyses have not revealed.

A. Rebound Effect

In [2] the rebound effect has been discovered for a four-node system: For delays $\tau > \tau_c$ ($8s$ when $T = 2s$) the system always is unstable. We investigate whether this effect persists if consumers are heterogeneous. We perform simulations for 10000 design points with the input values specified in the last column of Table II where we choose $\tau_j \in [0.5, 10]s$ and $T_j = 2s$. Our space of values includes the ones explored in [2] as a special case.

Fig. 2 is an excerpt of the decision tree obtained. The path to the second leaf from the left reads as follows: If $\min(\tau_j) < 2.1$ and $\text{avg}(\gamma_j) \geq 0.5$ and $\text{avg}(\tau_j) < 4.8$ and $\max(\tau_j) \geq 8$, then the system is stable. This means that, in a stable grid, a consumer may have a reaction time higher than $\tau_c \approx 8s$ as long as there is a consumer reacting quite fast, and the average reaction time is moderate. Moreover, the presence of a consumer reacting slowly has a positive effect on stability in this case. These results, stemming from our consideration of heterogeneity of participants and discovered with our approach, are new – they have not been insinuated in [2] in particular.

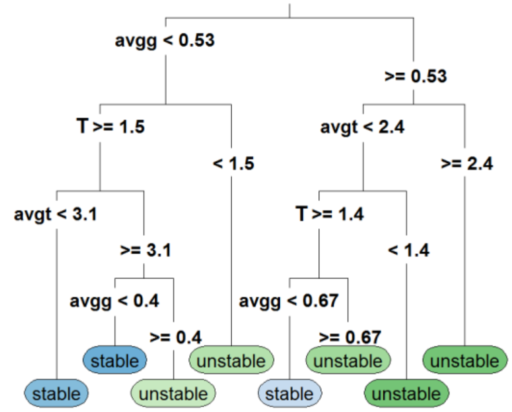

Fig. 3. The decision tree for system design.

B. Defining Values for Control Inputs

According to [2], there is a tradeoff between avoiding the rebound effect with small values of τ_j and increasing the basin stability with high values of τ_j; the suggested value is $\tau \approx 4$. At the same time, higher values of averaging time T have a positive effect on stability. We now check to what extent these conclusions hold for the system with diverse participants and sketch a way how our proposed approach could help choosing the values for control inputs. To do so, we allow one control input, T_j, to vary. We assume that $T_j = T$. This is a realistic assumption, since the averaging time can be inherent to the frequency-measuring device, and a policy maker can define it. We simulate the system for 10000 design points with the input values specified in the last column of Table II with $\tau_j \in [0.5, 5]s$ and $T = [0, 4]s$. As before, this includes the input values explored in [2] as a special case. Fig. 3 graphs the tree. First, the analysis confirms that high values of T are good for stability. Specifically, the stable leaves of the tree only exist for $T \geq 1.4$. This information can be used when designing the system, to, say, regulate averaging time, prohibiting values lower than 1.5. One can also see that all leaves classified as stable imply that $\text{avg}(\gamma_j) < 0.67$. In principle, one could also regulate this by adjusting the parameter connecting price to the grid frequency, c_1, in (7). But then the impact on demand response should be assessed. In other words, setting $c_1 = 0$ leads to perfect stability but completely cancels the effect of the economic superstructure.

Our results suggest that values of $\text{avg}(\tau_j)$ smaller than 3.1 contribute to system stability. For $\text{avg}(\tau_j) \geq 3.1$, the stable region (the second leaf), occupies only a small share of the design space. So $\tau \approx 4$ might not be a good value for a system with heterogeneous consumers. We observe that power does not appear in any tree. We speculate that it only has a small or even no impact on stability as long as the physical system is stable. So a takeaway is that the view on the system is more differentiated with the use of decision trees than in [2].

VI. Discussion

Although the method we used to study the results of simulations already works quite well, we see space for further generalization. This section contains aspects this study has not addressed which we deem promising research directions.

In of our experiments, accuracy (the share of correctly classified design points) has been around 80%. To obtain a general view on the system, this is not crucial, and our results remain valid. However, when one uses simulations to find pure stability regions, this may be unsatisfactory. Considering cost-sensitive classifiers, which put more emphasis on stable design points, is one possible direction of research.

Next, we have used the knowledge of system symmetry to obtain the features from the initial input data by computing aggregates. However, questions such as whether our features are optimal in some sense, or whether such features can be "discovered" automatically remain open.

Additionally, the extension of analysis for large grids, containing more than 10 participants, is not straightforward.

VII. Conclusions

Decentral Smart Grid Control, the topic of this article, has been touted as a way to realize demand response. We have collected the assumptions behind it systematically, some of which are restrictive. In order to eliminate some assumptions, while at the same time targeting at simple and insightful models, we have proposed the following:

- Simulate the system for diverse sets of input values. A simulation result has been an inference on whether the system is stable for the specific input values.
- Create features from original inputs, to reduce the number of degrees of freedom.
- Apply a decision-tree algorithm to the data resulting from all simulations.

This approach does reveal new insights regarding DSGC, not known from previous studies. For example, we have learned that the system can be stable even if some participants adapt their energy consumption with a high delay, or fast adaptation is preferable for stability under certain conditions.

References

[1] B. Schäfer, M. Matthiae, M. Timme, and D. Witthaut, "Decentral smart grid control," *New Journal of Physics*, vol. 17, no. 1, p. 15002, 2015.

[2] B. Schäfer, C. Grabow, S. Auer, J. Kurths, D. Witthaut, and M. Timme, "Taming instabilities in power grid networks by decentralized control," *European Physical Journal: Special Topics*, vol. 225, no. 3, pp. 569–582, 2016.

[3] J. P. C. Kleijnen, *Design and analysis of simulation experiments*, vol. volume 230. 2015.

[4] A. A. Freitas, "Comprehensible Classification Models – a position paper," *ACM SIGKDD Explorations Newsletter*, vol. 15, no. 1, pp. 1–10, 2013.

[5] J. Huysmans, K. Dejaeger, C. Mues, J. Vanthienen, and B. Baesens, "An empirical evaluation of the comprehensibility of decision table, tree and rule based predictive models," *Decision Support Systems*, vol. 51, no. 1, pp. 141–154, 2011.

[6] M. R. Berthold and K. P. Huber, "Constructing fuzzy graphs from examples," *Intelligent Data Analysis*, vol. 3, no. 1, pp. 37–53, 1999.

[7] H. Pierreval, "Rule-based simulation metamodels," *European Journal of Operational Research*, vol. 61, no. 1–2, pp. 6–17, 1992.

[8] T. Yoshida and S. Nakasuka, "A dynamic scheduling for flexible manufacturing systems: hierarchical control and dispatching by heuristics," in *Proceedings of the 28th Conference on Decirion and Control*, 1989.

[9] H. Pierreval and H. Ralambondrainy, "A Simulation and Learning Technique for Generating Knowledge about Manufacturing Systems Behavior," *J. Opl Res. Soc*, vol. 41, no. 6, pp. 461–474, 1990.

[10] J. D. McCajiey, Q.-L. Q. Zhao, S. Wang, G.-Z. G. Zhou, R. T. R. T. Treinen, J. D. McCalley, S. Wang, Q.-L. Q. Zhao, G.-Z. G. Zhou, R. T. R. T. Treinen, and A. D. Papalexopoulos, "Security boundary visualization for systems operation," *IEEE Transactions on Power Systems*, vol. 12, no. 2, pp. 940–947, May 1997.

[11] B. Jayasekara and U. D. Annakkage, "Derivation of an accurate polynomial representation of the transient stability boundary," *IEEE Transactions on Power Systems*, vol. 21, no. 4, pp. 1856–1863, 2006.

[12] L. S. Moulin, A. P. A. Silva, S. Member, and R. J. M. Ii, "Support Vector Machines for Transient Stability Analysis of Large-Scale Power Systems," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 818–825, 2004.

[13] T. Amraee and S. Ranjbar, "Transient instability prediction using decision tree technique," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 3028–3037, 2013.

[14] G. Filatrella, A. H. Nielsen, and N. F. Pedersen, "Analysis of a power grid using a Kuramoto-like model," *European Physical Journal B*, vol. 61, no. 4, pp. 485–491, 2008.

[15] M. Rohden, A. Sorge, M. Timme, and D. Witthaut, "Self-organized synchronization in decentralized power grids," *Physical Review Letters*, vol. 109, no. 6, 2012.

[16] A. Ulbig, T. S. Borsche, and G. Andersson, "Impact of low rotational inertia on power system stability and operation," in *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 2014, vol. 19, pp. 7290–7297.

[17] D. Manik, D. Witthaut, B. Schäfer, M. Matthiae, A. Sorge, M. Rohden, E. Katifori, and M. Timme, "Supply networks: Instabilities without overload," *European Physical Journal: Special Topics*, vol. 223, no. 12, pp. 2527–2547, 2014.

[18] P. S. Kundur, *Power System Stability and Control*. McGraw-Hill, 1994.

[19] W. W. Price, C. W. Taylor, G. J. Rogers, K. Srinivasan, C. Concordia, M. K. Pal, K. C. Bess, P. Kundur, B. L. Agrawal, J. F. Luini, E. Vaahedi, and B. K. Johnson, "Standard Load Models for Power Flow and Dynamic Performance Simulation," *IEEE Transactions on Power Systems*, vol. 10, no. 3, pp. 1302–1313, 1995.

[20] T. Nishikawa and A. E. Motter, "Comparative analysis of existing models for power-grid synchronization," *New Journal of Physics*, vol. 17, no. 1, p. 15012, 2015.

[21] K. Schmietendorf, J. Peinke, R. Friedrich, and O. Kamps, "Self-organized synchronization and voltage stability in networks of synchronous machines," *The European Physical Journal Special Topics*, vol. 2592, pp. 2577–2592, 2014.

[22] O. Kamps, "Characterizing the Fluctuations of Wind Power Production by Multi-time Statistics," *Wind Energy-Impact of Turbulence*, pp. 67–72, 2014.

[23] M. D. Morris, "The Design and Analysis of Computer Experiments," *Journal of the American Statistical Association*, vol. 99, no. 468, pp. 1203-- 1204, 2004.

[24] M. G. Lijesen, "The real-time price elasticity of electricity," *Energy Economics*, vol. 29, no. 2, pp. 249–258, 2007.

[25] L. Breiman, *Classification and Regression Trees*. Routledge, 2017.

[26] T. W. Simpson, J. D. Poplinski, P. N. Koch, and J. K. Allen, "Metamodels for Computer-based Engineering Design: Survey and recommendations," *Engineering with Computers*, vol. 17, no. 2, pp. 129–150, 2001.

[27] G. G. Wang and S. Shan, "Review of Metamodeling Techniques in Support of Engineering Design Optimization," *Journal of Mechanical Design*, vol. 129, no. 4, p. 370, 2007.

[28] M. Stein, "Large Sample Properties of Simulations Using Latin Hypercube Sampling," *Technometrics*, vol. 29, no. January 2014, pp. 37–41, May 2012.

[29] R. T. Johnson, D. C. Montgomery, and K. S. Kennedy, "Hybrid space-filling designs for computer experiments," in *Frontiers in Statistical Quality Control 10*, 2012, pp. 287–301.